

# Heavy quark diffusion in a hydrodynamically expanding medium

A Langevin dynamical calculation

Andrew M. Adare<sup>a</sup>  
Michael P. McCumber<sup>a,b</sup>  
Jamie L. Nagle<sup>a</sup>

with thanks to

Paul Romatschke<sup>a</sup>

<sup>a</sup>University of Colorado, Boulder

<sup>b</sup>Los Alamos National Laboratory

29th Winter Workshop on Nuclear Dynamics  
Squaw Valley, CA  
February 7, 2013

# Heavy quarks in the QGP

A well-calibrated probe of the medium

- They are distinct
  - $c, b$  are **conserved quantum numbers**
  - Identifiable from final-state products
- They are around at the beginning
  - But they **thermalize slowly**:  
 $\tau \approx M/T \approx 6\times$  longer than for light quarks
  - Well-defined initial conditions (hard processes)
- They suffer **collisional energy loss**
  - $c$  vs  $b$  energy loss an important constraint
  - Diffusion
  - Drag

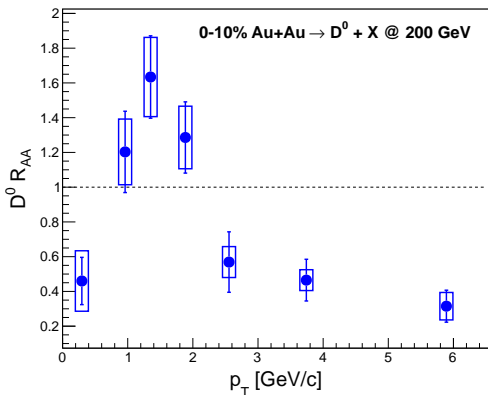
# $D^0$ modification experimentally established

Not a weak effect!

Initial naive thinking (large mass  $\Rightarrow$  small modification) is unsupported.

## STAR $D^0$ s in Au+Au

STAR Preliminary (QM 2012)



- Features in data:
  - Strong low- $p_T$  suppression
  - Large enhancement around 1-2 GeV
  - 2-3 $\times$  suppression for  $p_T > 2$  GeV

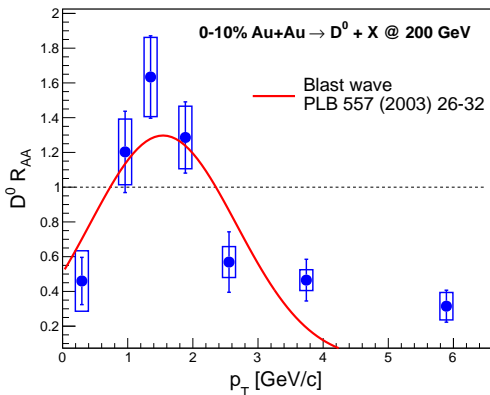
# $D^0$ modification experimentally established

Not a weak effect!

Initial naive thinking (large mass  $\Rightarrow$  small modification) is unsupported.

## STAR $D^0$ s in Au+Au

STAR Preliminary (QM 2012)



- **Blast-wave fit** (Batsouli, Kelly, Gyulassy, Nagle) describes low- $p_T$  data well
- **But** with boosted thermal particles only, the high- $p_T$  behavior is not matched.

The radial velocity

$$\beta_T = \beta_{\max} \frac{r}{R}$$

has a linear boost profile, with uniform initial density.



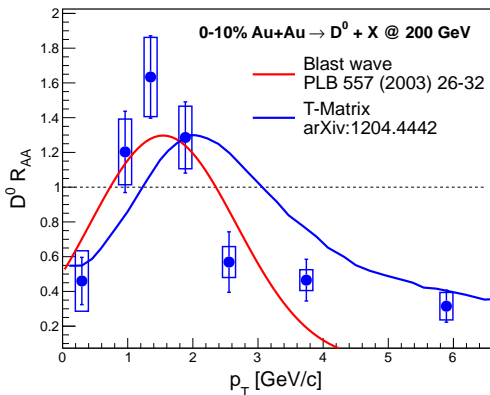
# $D^0$ modification experimentally established

Not a weak effect!

Initial naive thinking (large mass  $\Rightarrow$  small modification) is unsupported.

## STAR $D^0$ s in Au+Au

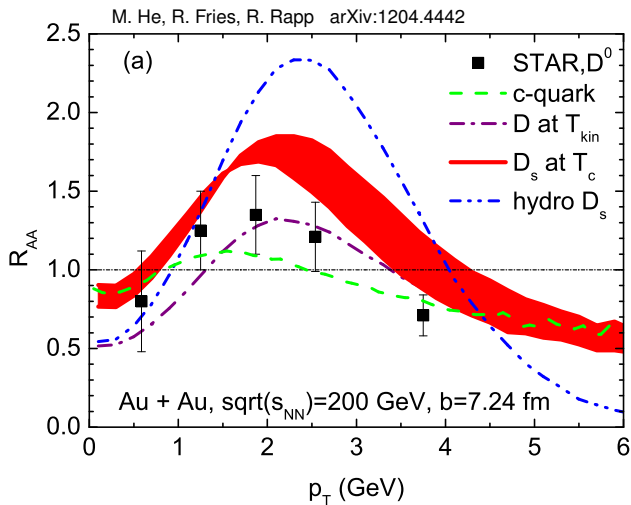
STAR Preliminary (QM 2012)



- T-matrix interactions + coalescence (M. He, R. Fries, R. Rapp) roughly captures  $R_{AA}$  features over full  $p_T$  range.
- Model includes a non-thermal component.
- How much of this shape comes from hadronization?

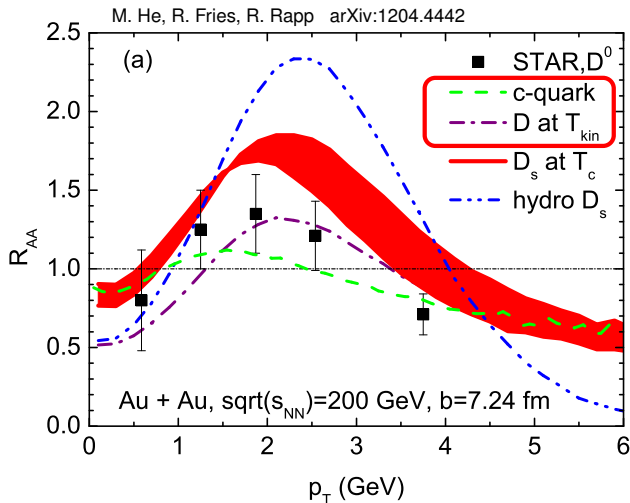
# Quark vs. hadron $R_{AA}$

Coalescence is an important effect!



# Quark vs. hadron $R_{AA}$

Coalescence is an important effect!



Compare **c-quark** and **D-meson**:

Far stronger low- $p_T$  suppression and intermediate- $p_T$  enhancement in hadronization stage

# Modeling heavy quark diffusion

Checking further into things

Since heavy quark modification is so informative, modeling in-medium interactions is valuable.

Many groups have produced interesting calculations. We throw our hat into the ring as well.

We used a Langevin MC model embedded in 2+1D viscous hydro by P. Romatschke.

# Modeling heavy quark diffusion

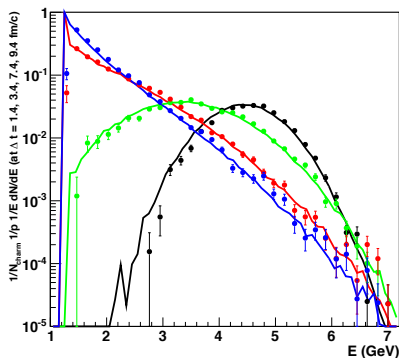
Checking further into things

Since heavy quark modification is so informative, modeling in-medium interactions is valuable.

Many groups have produced interesting calculations. We throw our hat into the ring as well.

We used a Langevin MC model embedded in 2+1D viscous hydro by P. Romatschke.

**Check:** for a static uniform medium,  $p_T$  spectra match an established calculation.



Phys. Rev. C 84, 064902 (2011)

# Modeling heavy quark diffusion

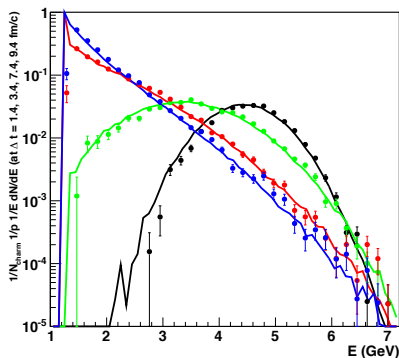
Checking further into things

**Check:** for a static uniform medium,  $p_T$  spectra match an established calculation.

Since heavy quark modification is so informative, modeling in-medium interactions is valuable.

Many groups have produced interesting calculations. We throw our hat into the ring as well.

We used a Langevin MC model embedded in 2+1D viscous hydro by P. Romatschke.



Phys. Rev. C 84, 064902 (2011)

Now for some details about the calculation. . .

# Langevin dynamics

## Brownian motion in a thermal bath

The **Langevin equation**:

$$\frac{d\mathbf{p}^i(t)}{dt} = -\eta_D^{ij}\mathbf{p}^j(t) + \xi^i(t)$$

- Viscous drag force  $\eta_D^{ij}$  describes large-scale average motion.
- $\xi^i$  describes stochastic fluctuations about the average motion

$$\langle \xi^i(t)\xi^j(t') \rangle = 4TE\eta_D^{ij}\delta(t-t'), \quad \langle \xi^i(t) \rangle = 0.$$

# Langevin dynamics

## Brownian motion in a thermal bath

The **Langevin equation**:

$$\frac{d\mathbf{p}^i(t)}{dt} = -\eta_D^{ij}\mathbf{p}^j(t) + \xi^i(t)$$

- Viscous drag force  $\eta_D^{ij}$  describes large-scale average motion.
- $\xi^i$  describes stochastic fluctuations about the average motion

$$\langle \xi^i(t)\xi^j(t') \rangle = 4TE\eta_D^{ij}\delta(t-t'), \quad \langle \xi^i(t) \rangle = 0.$$

### Time discretization

In each step  $\Delta t$ , the quark suffers a normally-distributed random deflection:  $\sigma = \sqrt{2T^2/(D\Delta t)}$ , where  $D$  is the diffusion parameter.

One parameter controls essential physics (scattering, drag, boosts)

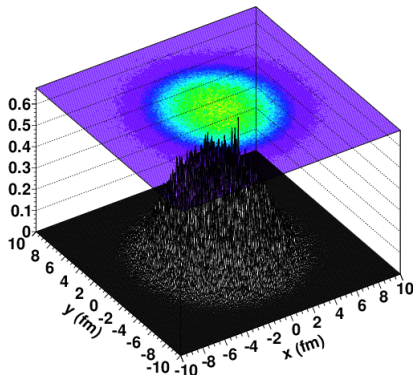


# Langevin + Hydro inputs

## Initial conditions

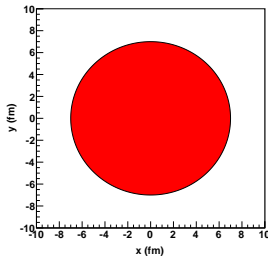
Initial quark positions distributed according to MC Glauber  $N_{coll}$  distribution

Glauber Au+Au  $N_{coll}$  distribution  $b = 2.0$  fm



Contrast with linear boost profile of blast-wave:

Effective density for blast-wave model



Many particles at large radii  
 $\Rightarrow$  high sensitivity to late-stage expansion

# Langevin + Hydro inputs

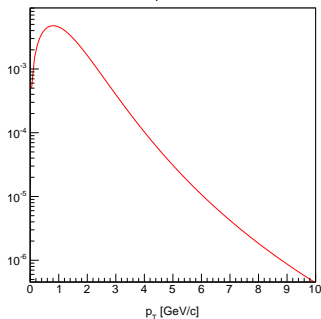
## Quark momentum distributions

Charm quark momentum distributions are given this shape:

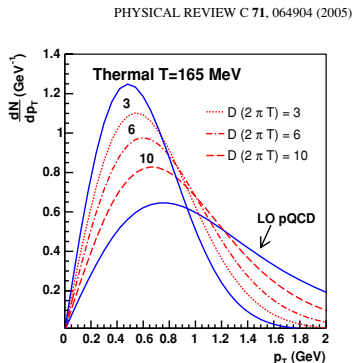
$$\frac{1}{p_T} \frac{dn}{dp_T} \propto \frac{1}{(p_T^2 + \Lambda^2)^\alpha}$$

where  $\alpha = 3.9$ ,  $\Lambda = 2.1$ , following Cao, Qin, & Bass (arXiv:1205.2396v1)

Charm  $p_T$  distribution



This form also used by Moore & Teaney (labeled "LO pQCD"):



# Langevin MC simulation

8  $c\bar{c}$  pairs shown

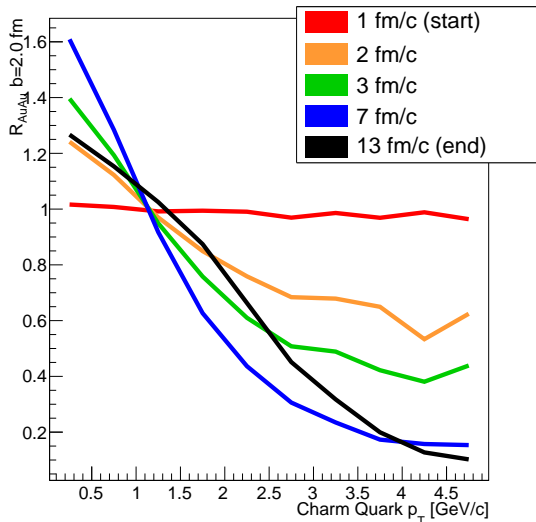
# Langevin MC simulation

$D$  parameter corresponding to  $\eta/s = 1/4\pi$

# $R_{AA}$ : time dependence

subtitle

$R_{AA}$  for  $\eta_D/s = 1/4\pi$ : Time dependence



Increasing high- $p_T$  suppression with time...

but  $R_{AA} < 1$ .

Late hydro push ( $t > 7$  fm/c) decreases low- $p_T$  contribution, but not enough to make  $R_{AA} < 1$ .

# $R_{AA}$ : varying the effective coupling

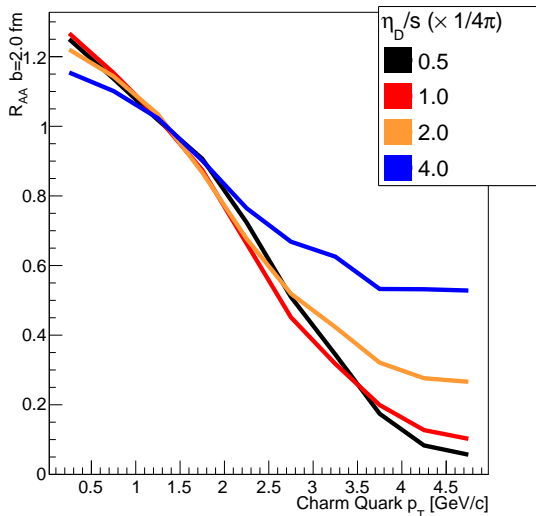
Smaller  $\eta/s \Leftrightarrow$  larger diffusion

Even after varying the diffusion by an order of magnitude, the **low- $p_T$   $R_{AA}$  changes weakly.**

$R_{AA} > 1$ , no matter the  $D$  value.

**Cancellation** over time:  
Drag at early times + boost at late times

## Charm quark $R_{AA}$



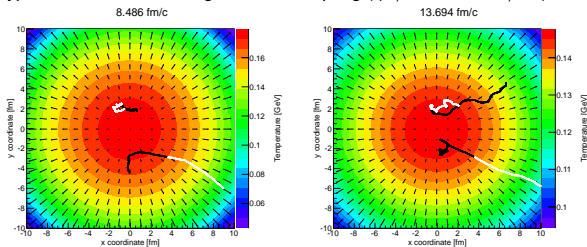
# Azimuthal $q\bar{q}$ correlations

Seeking a more sensitive measure of early-time dynamics

Initial  $q\bar{q}$  pairs given equal and opposite  $p_T$

Strong effective coupling (small  $D$ ): expect small-angle correlations from late-stage boosts

Typical behavior for strong effective coupling ( $\eta_D/s = 0.5 \times 1/4\pi$ ):



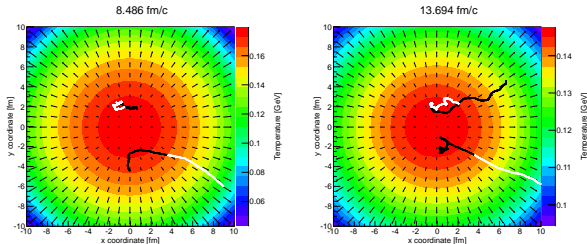
# Azimuthal $q\bar{q}$ correlations

Seeking a more sensitive measure of early-time dynamics

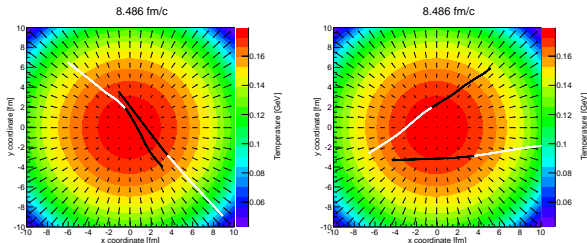
Initial  $q\bar{q}$  pairs given equal and opposite  $p_T$

Strong effective coupling (small  $D$ ): expect small-angle correlations from late-stage boosts

Typical behavior for strong effective coupling ( $\eta_D/s = 0.5 \times 1/4\pi$ ):



Typical behavior for weak effective coupling ( $\eta_D/s = 4 \times 1/4\pi$ ):



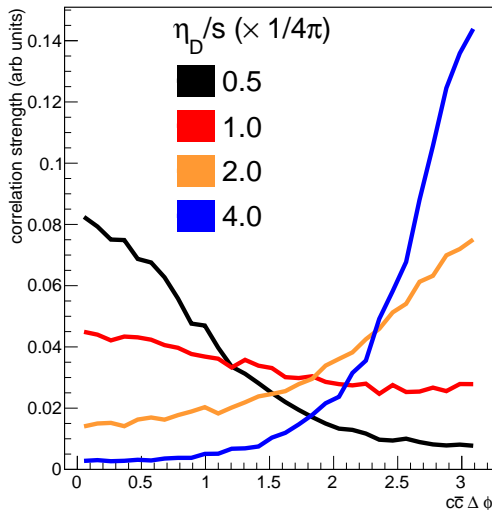
Large  $D$ : back-to-back correlation is preserved



# How does $\Delta\phi$ depend on effective coupling?

$\Delta\phi$  distribution for  $p_T > 1 \text{ GeV}/c$

$c\bar{c} \Delta\phi$  distribution



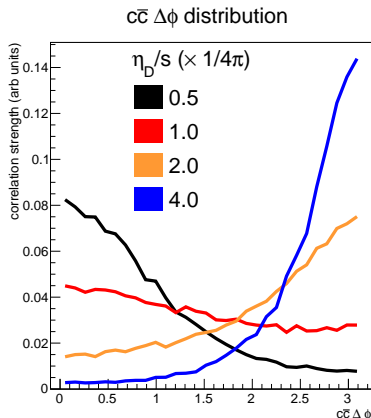
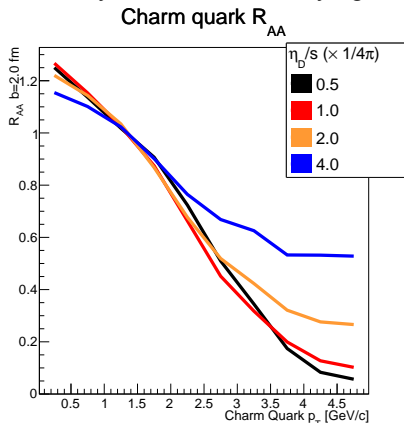
**Strong coupling** (small  $D$ )  
enhances same-side  
correlations

**Weak coupling** (large  $D$ )  
corresponds to less deflection,  
preserving back-to-back  
production.

# $R_{AA}$ and $\Delta\phi$ vs diffusion strength

Comparison of “observables”

$R_{AA}$  is fairly insensitive to varying  $D$  near expected values.



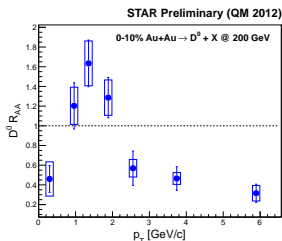
But the  $c\bar{c}$  correlation function changes dramatically from same-side  $\rightarrow$  opposite-side dominance.

# Understanding the shape of $R_{AA}$

Especially, low- $p_T$  suppression

Despite large range of coupling parameters tried, we could not get  $R_{AA} < 1$  below  $\approx 1$  GeV/ $c$ .

- Perhaps the low  $p_T$  effect is all coalescence?
- Or perhaps there is something to make the physics for  $t < 7$  fm/ $c$  weaker, while preserving the strong coupling for  $t > 7$  fm/ $c$ .

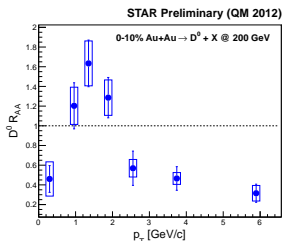


# Understanding the shape of $R_{AA}$

Especially, low- $p_T$  suppression

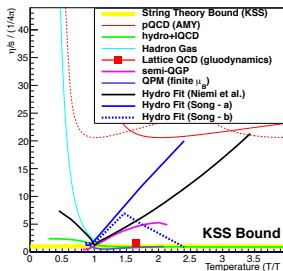
Despite large range of coupling parameters tried, we could not get  $R_{AA} < 1$  below  $\approx 1$  GeV/c.

- Perhaps the low  $p_T$  effect is all coalescence?
- Or perhaps there is something to make the physics for  $t < 7$  fm/c weaker, while preserving the strong coupling for  $t > 7$  fm/c.



To check this, we investigate a temperature-dependent coupling.

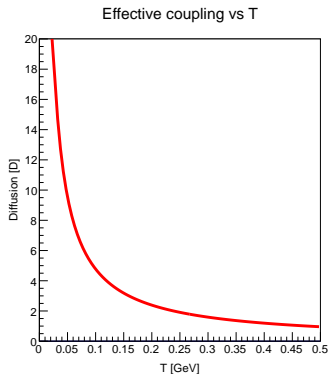
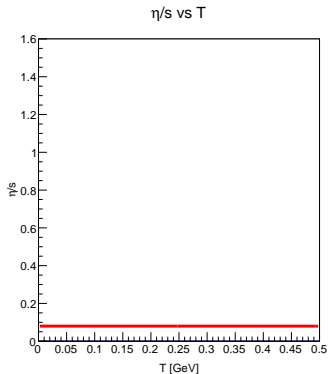
Figure from sPHENIX proposal  
arXiv:1207.6378



# Temperature-dependent $\eta/s$

and the corresponding effective coupling  $D(T)$

For constant  $\eta/s = 1/4\pi$ , the diffusion parameter is  $D = 3/2\pi T$ .



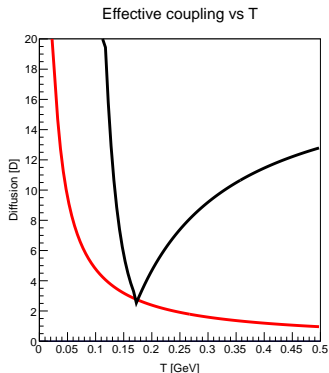
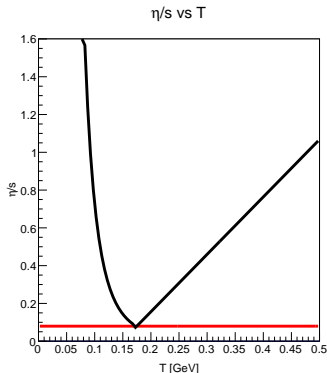
# Temperature-dependent $\eta/s$

and the corresponding effective coupling  $D(T)$

For constant  $\eta/s = 1/4\pi$ , the diffusion parameter is  $D = 3/2\pi T$ .

Based on many models,  $\eta/s$  grows sharply away from  $T_c$

$\Rightarrow D$  much larger at high  $T$  / early time.



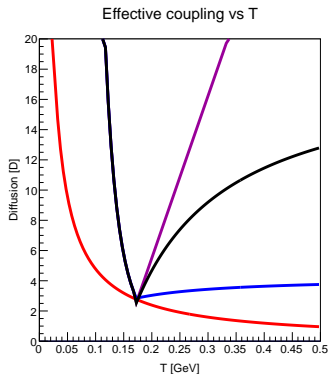
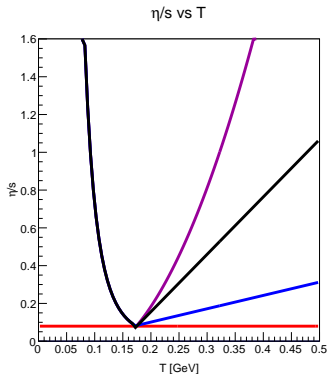
# Temperature-dependent $\eta/s$

and the corresponding effective coupling  $D(T)$

For constant  $\eta/s = 1/4\pi$ , the diffusion parameter is  $D = 3/2\pi T$ .

Based on many models,  $\eta/s$  grows sharply away from  $T_c$

$\Rightarrow D$  much larger at high  $T$  / early time.



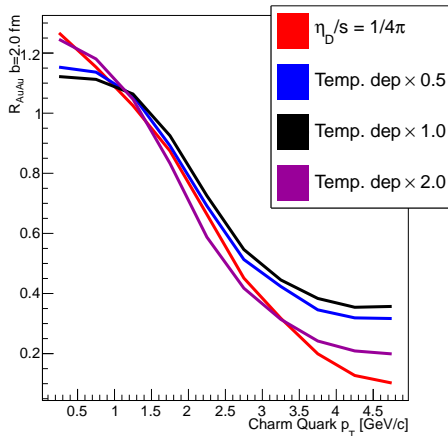
We also tried **doubling** and **halving** the high- $T$  dependence.  $\Rightarrow$  Result:

# $R_{AA}$ and $\Delta\phi$ for $D(T)$

## Temperature-dependent diffusion

$R_{AA}$  is fairly insensitive to the temperature dependence.

Charm quark  $R_{AA}$  : T-dependent  $\eta_D/s$

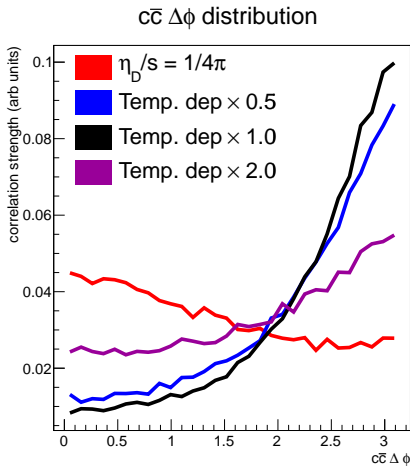
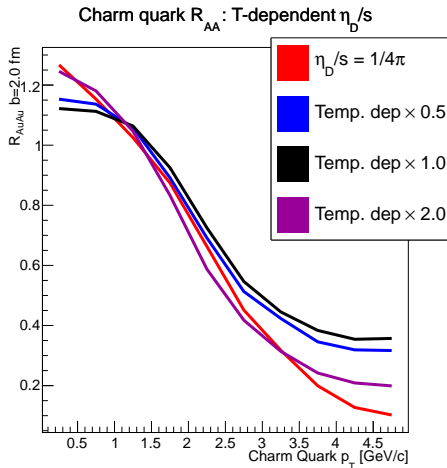




# $R_{AA}$ and $\Delta\phi$ for $D(T)$

## Temperature-dependent diffusion

$R_{AA}$  is fairly insensitive to the temperature dependence.



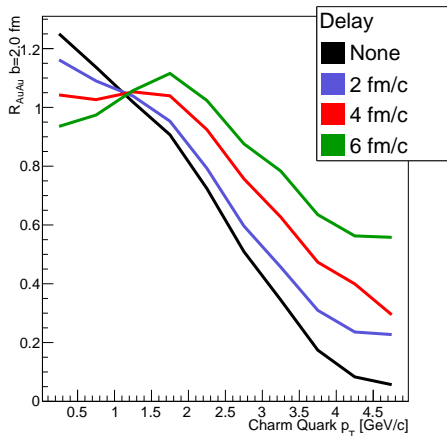
On the other hand, the pair correlation shows a strong dependence!

# Delayed diffusion

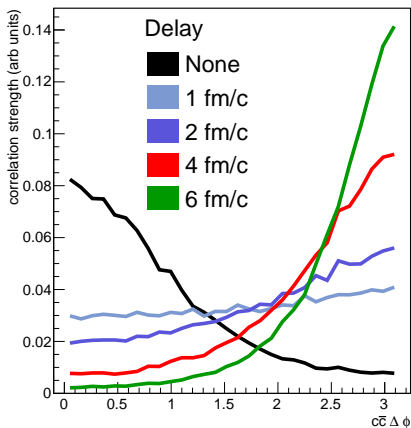
What if charm quarks don't interact at all initially?

**Immediate diffusion:** strong initial drag, then pairs are collinearized in late-stage boost

Charm quark  $R_{AA}$ : Start time dependence



$c\bar{c}$   $\Delta\phi$  distribution,  $D = 3/4\pi$



**Very late diffusion:** back-to-back enhancement as for large  $\eta/s$  case.

# Summary

## Lessons from this study

**Langevin:** simple & informative for quark-medium interactions for understanding QGP transport properties

- Scattering + viscous drag  $\rightarrow$  high- $p_T$  suppression
- Also causes low- $p_T$  enhancement

# Summary

## Lessons from this study

**Langevin:** simple & informative for quark-medium interactions for understanding QGP transport properties

- Scattering + viscous drag  $\rightarrow$  high- $p_T$  suppression
- Also causes low- $p_T$  enhancement

**But:** Very low- $p_T$  suppression doesn't come about at the partonic level

- Whatever we tried, we could not get  $R_{AA} < 1$
- With realistic initial geometry, late-stage boosts are insufficient
- What really does it is coalescence

# Summary

## Lessons from this study

**Langevin:** simple & informative for quark-medium interactions for understanding QGP transport properties

- Scattering + viscous drag  $\rightarrow$  high- $p_T$  suppression
- Also causes low- $p_T$  enhancement

**But:** Very low- $p_T$  suppression doesn't come about at the partonic level

- Whatever we tried, we could not get  $R_{AA} < 1$
- With realistic initial geometry, late-stage boosts are insufficient
- What really does it is coalescence

**Angular  $c\bar{c}$  correlations** are sensitive where  $R_{AA}$  is not, but of course we can't measure them directly

We experimentalists need to think hard about how to access this physics.

# Summary

## Lessons from this study

**Langevin:** simple & informative for quark-medium interactions for understanding QGP transport properties

- Scattering + viscous drag  $\rightarrow$  high- $p_T$  suppression
- Also causes low- $p_T$  enhancement

**But:** Very low- $p_T$  suppression doesn't come about at the partonic level

- Whatever we tried, we could not get  $R_{AA} < 1$
- With realistic initial geometry, late-stage boosts are insufficient
- What really does it is coalescence

**Angular  $c\bar{c}$  correlations** are sensitive where  $R_{AA}$  is not, but of course we can't measure them directly

We experimentalists need to think hard about how to access this physics.

**Thanks**